Chapter Two: The Pendulum and Standards of Measure in the Ancient World

Roland A. Boucher

When the French proposed their first metric system in 1723, they had no idea it had been invented by the ancient Mesopotamians 5000 years earlier. Just as the French proposed to use the length of a one-second pendulum to create standards of length, volume, and weight, the Sumerians created nearly identical meters, liters and kilograms. Our research shows that the Sumerians in ancient Mesopotamia used both the Moon and the Sun as their clock. It appears that the Egyptians improved on this timing accuracy by using the stars. Later the Minoans introduced the use of the planet Venus as a clock.

These concepts spread throughout the ancient world from Britain in the West to Japan in the East. The Minoan standards are immortalized in the Magna Carta of 1215. The old English saying "a pint a pound the world around" had been true for over 3000 years. In the 19th Century, both Stuart and Penrose accurately measured the dimensions of the Parthenon, finding its width to be 0.9997 arc seconds on the polar circumference of the Earth. This accuracy puzzled scholars for 150 years. Our research shows the width of the Parthenon in Athens was designed to be 1/30 of the perimeter of the Great Pyramid of Giza. The same Pendulum Formula, when timed with Venus rather than the Sun, increased the pendulum length just the right amount. This precision was not dumbfounding – it was just dumb luck.

Introduction

Chapter One, in the previous issue (24:1) of *CAL LAB: The International Journal of Metrology*, showed how five pendulums could have established five Sumerian standards of length in Ancient Mesopotamia. Precise matches were found among 32 of Dr. Powell's inscribed weights, among 3 matches of Sir Arthur Evans' Talent weights, and among 7 of Mr. Berriman's lengths, volumes and weights.

Chapter Two will examine additional standards from Egypt and the Minoan civilization on Crete. It will describe a special standard of length, volume, and weight developed in Sumeria, based on the polar circumference of the Earth. A case will be made that this new standard was used to establish the perimeter of the Great Pyramid of Giza at 30 Arc seconds of the polar circumference of the Earth. Two thousand years later, it was used to establish the width of the Parthenon at almost exactly one arc second.

The Standards of Ancient Egypt Circa 3000 B.C.E.

The Egyptians apparently realized that a star, a mere pinpoint of light, could provide a much higher level of precision than the Sun when measuring an interval of time as they developed their own standards. These standards appear to be based on the length of a Foot of approximately 300 mm. This Foot was developed with a pendulum which beat 366 times in the period it took the Earth to rotate through one 366th of a celestial day (one which is measured by the daily motion of a star). This pendulum length was used to create a Cable of 366 times twice the length of this pendulum. The length of the Egyptian Foot became 1/1000 of these Cable lengths.

The stars arrive at the same position in the sky about four modern minutes earlier each day due to the Earth's orbital motion, which may be why the Sumerians divided the day into four-minute intervals they called a Gesh. The star field appears to rotate 366 times in a year, so the number 366 was very important to an astronomer. The length of this new Cable was about 300 meters and their Djser (foot) about 300 mm. The Egyptians based all their measurement standards on their Djeser which was divided into 16 Egyptian Djeba or digits (18.75 mm). This digit was used to develop their Reman, Cubit, and Royal Cubit.

Mr. A.E. Berriman established the length of the Royal Cubit through the volume of Bowls #27 and #8 in the Petri collection at the University College, London [19], as follows:

Bowl #8 Volume = 366.2 cu in = 400 Ro = 1/16 Khar
3/2 Khar = **8789** cu in

Bowl #27 Volume = 546.5 cu in = 600 Ro = 1/16 Royal Cubic Cubit
Royal Cubic Cubit = **8744** cu in

Bowl #8 Royal Cubit = $\sqrt{\mathbf{8789}}$ cu in = 20.638 in = 524.21 mm
Foot = 299.55 mm

Bowl #27 Royal Cubit = $\sqrt{\mathbf{8744}}$ cu in = 20.602 in = 523.28 mm
Foot = 299.02 mm

Note: The volume of the Ro = 1 cubic finger or 1/9600 Khar.

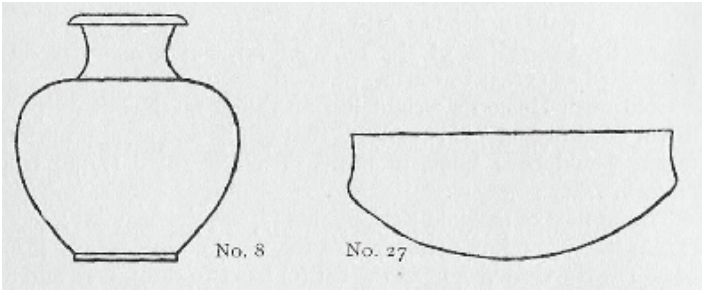

Figure. 8 Egyptian Bowls #8 and #27 in the Petrie collection.

Egyptian Old Kingdom nominal standards of measure:

- Standard of length #1 = Djser (foot) = (366 pendulum lengths)/1000 = (300 mm)
- Standard of length #2 = Reman = 20/16 Djser (375 mm)
- Standard of length #3 = Cubit = 24/16 d=Djser (450 mm)
- Standard of length #4 = Royal Cubit = 28/16 Djser (525 mm)
- Standard of volume = Khar = Volume of 2/3 of the Royal Cubit Cubed (96.5 liters)
- Standard of weight = Deben = 13.6 gm = 3 Sumeria or 2 Minoan Gold Standards

Detail of Calculations for the Egyptian Foot

The Egyptian pendulum beat 366 times in the period of 1/366 celestial day or 235.721 seconds. A Cable of 366 of these pendulum lengths established 1000 Egyptian feet. In Table 7-A we establish the theoretical length of this simple pendulum, then applying modest corrections for the period and length of a real pendulum, we develop the Egyptian Foot and Royal Cubit. In Table 7-B we establish the length of the Finger, Foot, Reman, Cubit, and Royal Cubit along with related volumes. Later in the New Kingdom, the Khar was reduced to 78.6 liters. In Table 7-C, we show standards of weight for both the New and Old Kingdoms.

Pendulum 5	Length mm	Cable m	Matching Values	
P = 0.64405 sec	820.76	300.4	1/360 deg = 307.701m @ Luxor WGS 84	
P - 0.15%	818.3	299.5	5000 Reman = 1.0139 arc minutes @ Luxor	
Foot, mm	299.5	NA	299.5 mm	A. E. Berriman [19]
Royal Cubit	524.10	NA	524.2 mm	A. E. Berriman [19]

Table 7-A. The length of the Egyptian Pendulum, Cable, Foot, and Royal Cubit.

Pendulum 5	R	Length mm	Volume liter	Name	Ratio	Volume liter
Royal Cubit	28	524.1	143.987	Deny	1	144
Cubit	24	449.3	90.674	Khar	2/3	96
Reman	20	374.4	52.473	Heqat	1/30	4.8
Foot	16	299.5	26.865	Hinu	1/300	0.48
Finger	1	18.719	6.559 ml	Ro	1/9600	15 ml

Table 7-B. Egyptian Old Kingdom Lengths and Volumes.

Pendulum 5	R	Old Kingdom	New Kingdom
Sep	10	136 grams	910 grams
Deben	1	13.6 grams	91 grams
Kite	1/10	1.36 grams	9.1 grams

The Old Kingdom Deben = 3 Sumerian or 2 Minoan Gold Standards. In the New Kingdom, the Deben became 1/1000 Khar of water (91 gm).

Table 7-C. Egyptian Weight Standards for both Old and New Kingdoms.

The Minoan Foot and Its Cousins

Venus was an important goddess to the Minoans (2700-1100 BCE). They timed their pendulum from Venus while in opposition for 366 beats during the time it took Venus to divide the rotation of the Earth by 366. The planet Venus is closer to the Sun than the Earth and orbits the Sun in 244 days. By viewing Venus when it is on the opposite side of the Earth, its motion cancels out some of the apparent motion caused by the spinning Earth, lengthening the period for 1/366 Venus day to 236.504 seconds. This essentially divided the celestial solar day into 365.25 parts. The length of the resulting Cable was 303.6 meters and the Foot 303.6 mm. The elevation of the North Star on its daily circle around the Pole would now change one arc-minute for every 6090 feet the observer moved in a north-south direction. The Minoan Foot [22] was no more accurate in predicting latitude than the Egyptian Foot, but it seems to have traveled widely.

- Minoan standard of length: Foot = 303.6 mm.
- Minoan standard of volume: Amphora = 1 Cubic Foot = 27,984 cm^3.
- Minoan standard of weight: Talent[1] = 27,901 grams = the weight 1 Amphora of rain water at 20 °C.

As we have shown, the Ancient Sumerians divided their volume standard of one Amphora either by 60 or by halves yielding volumes of 1/8 and 1/64 Amphora:

The standard volume for a Pint of 1/64 Amphora = 437.25 cu cm.

The standard weight for a Pound = the weight of one Pint of rain water at 20 °C = 436.0 gm.

The standard weight of a Troy pound became = 1/60 Amphora of wheat at 0.8 kg/liter = 373.2gm.

Traveling from Crete to ancient Britain, we find the length of the Minoan pendulum in the length of Megalithic Yards in Stonehenge [23].

The Minoan Foot was Immortalized in the Magna Carta

The Minoan Foot established the length of an ancient English foot used to develop standards of volume for the Gallon, the Bushel and the London Quarter. These were documented in the Magna Carta in 1215 [24]. The Winchester Bushel was simply the volume of one Minoan Cubic Foot and the Pint 1/64 of this volume. The accuracy of these measurements would suggest that the English Mercantile Pound and Scottish Pound had been established precisely by the weight of one Minoan Pint of rain water. It also would appear that the Troy Pound had been established precisely by the weight of 1/60 of a bushel (Amphora) of wheat at 0.8 kg/liter.

Moving to Japan, we find the Japanese Shaku of 303.0 mm; a very close match to the Minoan Foot when taking into account the difference in latitude. The Japanese, just as the Minoans, were a maritime nation. It is interesting that the largest linear standard of Ancient Japan was the Ri of 12960 Shaku, a length almost exactly that of 1/10,000 the polar circumference of the Earth.

Detail of Calculations for the Minoan Foot

The Minoan pendulum beat 366 times in 236.504 seconds. The length of their Cable of 366 pendulum lengths was equal to 1000 Minoan Feet. In Table 8-A (on the following page), we establish the theoretical length for a simple pendulum and the resulting foot. Applying modest corrections for the period and length of a real pendulum, results in the following Foot, Sila, and Mina along with corresponding measured values from reliable sources.

The Minoan Foot in Early England

In Table 8-B we establish the Minoan Cubic Foot as an English Bushel, and its division into Gallon, Pint, as well as its Cubic Finger which established the Minoan Gold Standard. The English values were guaranteed by the Magna Carta of King John on June 15, 1215.

The Minoan Cubic Foot as a Talent and Its Mina of Grain

In Table 8-C, we establish the Talent as the weight of a Minoan Cubic Foot of water, as well as measured values. We were quite surprised to find that the Troy Pound was of Minoan origin.

The Magnificent Octopus Talent of Bagdad and the Polar Circumference of the Earth

The Octopus Talent was discovered in Knossos, Crete in 1901 by Sir Arthur Evans, who also indicated that it appeared to be of Babylonian origin. This magnificent 29,000 gram Talent Weight from circa 1650 B.C.E. may well have been commissioned to celebrate the 1000th anniversary of the building of the Great Pyramid at Giza. A quick calculation revealed that an Amphora filled with 29,000 gm of water at 20 °C would have a volume of 29,086 ml or the volume of a 307.54 mm cube. This length is within 0.45 mm of the geodetic foot at the latitude of Lagash.

A search for a simple modification of one of the Sumerian pendulums, which would provide a match in length, quickly produced results. Pendulum 3, which beat 360 times in 240 seconds, had produced the 318.56 mm Zhou Market Foot in China and the Fuss in Bern, Austria.

1 A 27,900 gram Bronze Talent was found at Knossos by Professor Halbherr in 1903 [8].

Pendulum 6	Length mm	Sila ml	Mina gram	Measured Values
P = 0.64619 sec	829.160	570.05	284.184	Calculated for a Simple pendulum
Foot	303.473	NA	NA	Calculated for a Simple pendulum
P + 210 ppm	303.60	571.08	284.70	Ratio ball/string, 210 ,swing, 1/20 L
Cubic Foot	303.60	27,984	27,901	303.6 mm A.E.Evans At Knossos [8]
Foot	303.64	NA	NA	Early English Foot [22]
Foot	303.09	NA	NA	303 mm = Japanese Shaku [25]

Table 8-A. Lengths(mm), Sila(milliliter), Mina (grams) and matching values.

Pendulum 6	Ratio	Length mm	Volume ml	Weight g	Measured
Bushel	1	303.60	27,984	27,901	Winchester Bushel [6]
Gallon	1/8	153.61	3,498	3,488	Wine Gallon [6]
Pint	1/64	76.805	437.25	435.96	Wine Pint [6]
Pint	1/64	76.805	437.25	435.96	437.4g Mercantile pound [6]
cu finger	1/4096	1.897	6.831	6.811	6.8 g = Minoan gold standard [7]

Table 8-B. The Minoan Cubic Foot as the Early English Winchester Bushel. The foot (mm), Bushel (cu cm), weight (grams) along with Matching Values (period + 210 ppm).

Pendulum 6	R	Weight, g	Measured
Talent	60	27,901	27900 g Bronze Talent #4 Halbherr, Crete [8]
Sila of water	1	465.02	465.004 g #72 1/2 mina Zeriya [7]
Sila of grain	1	372.02	373.241 g English Troy Pound Zupko [26]

Table 8-C. The Minoan cubic foot as a Talent divided into Mina (period + 210 ppm).

If Pendulum 3 were allowed to beat 366 rather than 360 times in 240 seconds, the length of the resulting Foot would be 307.23 mm. This new Pendulum 7 would be too short to time easily, but one eight feet long would work well. It would beat 150 times in 1/366 solar day (236.065 seconds). The length of an 8 foot version of Pendulum 7, when swung 10 degrees to each side with a Ball/String ratio of 100, would be 246.032 cm.

The resulting Cable of 307.54 meters is almost perfectly equal to 1/360 of a degree at the latitude of Lagash. It is only 0.146 percent short of the true value. The length of 100 of these new Sumerian feet is almost exactly 1 arc second on the Polar Circumference of the Earth.

The Perimeter of the Great Pyramid is within 0.25% of 30 arc seconds of the Polar Circumference of the Earth. This accuracy has puzzled scholars for almost 150 years.

The Great Pyramid of Giza was accurately measured by both Petrie and Cole, establishing the average width at 230.355 meters with a precision of better than one part in 10,000. The four sides are aligned north-south and east west to within 1/15 degree of the true values. The length of the four sides or perimeter is 921.421 meters is within 0.25% of 30 arc seconds (1/120 degree) of the Polar Circumference of the Earth established through satellite measurements.

Figure 9. Octopus Weight

Figure 10. Octopus Amphora

The Perimeter of the Great Pyramid of Giza was Established as 3000 Sumerian Feet

Sometime before 2680 B.C.E., when the construction of the Great Pyramid began, the Egyptian astronomers and engineers would have become aware that a Sumerian geodetic pendulum provided a very accurate measurement of the length of an arc-minute of latitude. Using Pendulum 7, to establish the perimeter of the Great Pyramid, provides a perfect match to the measured values with a modest 67 ppm correction for a physical pendulum when it is operated at Luxor. Past claims for a width of 440 Royal Cubits would require a rather small value for the Egyptian Foot as shown in Table 9.

Figure 11. The Great Pyramid at Giza Constructed in 2600 BCE.

The Egyptians may not have known just how accurate the Sumerian measurements were, but they were much better than the 1.4 percent error resulting from a 5000 Reman Nautical Mile.

The Mysterious Precision in the Construction of the Parthenon

The Parthenon in Athens, Greece, was accurately measured by Stuart in 1750 and later by Penrose in 1888 [17]. The dimension of the width of the Parthenon at 30.861 m appeared to be almost exactly one average arc second on the polar circumference of the Earth, 30.870 m [21]. The small 9 mm error out of 30870 mm was surprising considering that the true measure of the Earth was obtained in 1984 with satellite data. This level of accuracy was just not possible in 600 B.C.E.

The Octopus Talent Yields the Attic Foot in Athens

The accuracy with which the Attic Foot predicts the Polar Circumference of the Earth has perplexed scholars for 150 years. This extreme accuracy was simply the product of luck. Table 10-A (on the following page) shows the evolution of Pendulum 7 into the Octopus Talent found in Knossos. When it was timed using Venus rather than the Sun, Pendulum 7 lengthened about 0.37 percent, eliminating almost all error. Making minor correction for the properties of a real pendulum gave us the famous Attic foot.

The Octopus Talent, Amphora and Foot were Adopted by the Etruscans

It would appear that the Octopus Talent of 29000 grams found by Sir Arthur Evans in 1901 was the basis of the Etruscan measures of volume and weight. The Etruscan Wool Pound of 453.074 grams or 6992 grains is 1/64 of their Talent. It was selected by Queen Elizabeth I as a prototype for the 7000 grain British Imperial Pound.

The Greek Stadion and the Roman Foot

The Greeks created the Stadion of 600 Greek Feet. The length of this Stadion was one-tenth of a British Nautical Mile (600 Stadion = 1 degree on the polar circumference of the Earth [16]). The length of the Stadion was adopted by Rome as the Stadia. However, it contained 625 Roman Feet [28]. This made the length of the Roman Foot 296.296 mm.

The Romans also created a Mile of 8 Stadia or 5000 Roman Feet (1481.424 meters). Thus, there were 75 Roman Miles per degree of latitude and 27,000 Roman Miles in the

Pendulum 7	Foot	3000 Feet	Description
Gravity= 9.7943594	307.234 mm	921.702 m	simple pendulum 7 in Lagash
Gravity= 9.7900450	307.099 mm	921.297 m	simple pendulum 7 in Luxor
period + 67 ppm	307.140mm	921.421 m	921.421 meters Petrie & Cole
440 Royal Cubits	299.162 mm?	921.421 m	3080 Feet @ 299.162 mm

Table 9. The Great Pyramid was designed in conformance with Sumerian Pendulum 7.

Pendulum 7	Foot, mm	Talent, gm	Description
Gravity = 9.7943594	307.234 mm	28,915	simple pendulum 8 in Lagash
Gravity = 9.7975933	307.335 mm	NA	simple pendulum 8 in Knossos
+ 0.1% correction	307.535 mm	29,000 g	Octopus Talent in Knossos*
Gravity = 9.7999303	307.409 mm	NA	Simple pendulum 8 in Athens
Gravity = 9.7999303	308.553 mm	NA	Venus pendulum 8 in Athens
185 ppm correction	308.610 mm	NA	Measured value of Attic Foot **

** Correction for pendulum with 10 degree swing, ball/string ratio = 100*
*** Correction for pendulum with 10 degree swing ,2.4 cm Granite ball & 0.60 gm string*

Table 10-A. Evolution of the Attic Foot and Talent derived from Pendulum 7.

Pendulum 7	R	Volume ml	Weight gm	Measured
Amphora	64	29,086	29,000	307.535 mm cube of water @ room temp.
Gallon	8	3,635.75	3,625.00	no match
Pint	1	454.47	453.13	453.074 = Etruscan Wool Pound [18]
Pint	1	454.47	453.13	453.592 = British Imperial Pound [26]

Table 10-B. The Octopus Talent as Etruscan Amphora, Gallon, Pint, and Pound.

Polar Circumference of the Earth. We may never know if the Romans were aware of the accuracy with which their mile could measure the Earth.

Today, using modern satellite data, we find the circumference was eight Roman Miles short, an error of only 0.02 percent. The Romans used the ratio of (25:24) in developing their new Foot which would lead to cultures throughout Europe adopting it to other standard feet as well. The resulting confusion and profusion of European standards provided a strong impetus for reform.

Conclusion

In Chapter Two we have established three pendulum lengths which produced four Egyptian and two Sumerian standard lengths as well as the Greek Attic Foot, Stadia, Roman Foot and Roman Mile. Precise matches were found among 10 of Dr. Powell's inscribed weights, 4 matches among Sir Arthur Evans' lengths and Talent weights, and 2 among Mr. Berriman's lengths, volumes, and weights. A side trip to early England established Minoan roots in the Winchester Bushel and in both the Mercantile and Troy Pounds. A side trip to Japan established Minoan roots in early Japanese Standards showing that their longest standard of length, the Ri, was 1/10,000 the polar circumference of the Earth. It was quite a surprise to find that the both British Imperial and US Pounds are related to the Polar Circumference of the Earth. If you are in doubt, calculate the length of the edge of a 64 million pound cube of water at room temperature and compare it to one arc second of the Polar Circumference of the Earth.

In conclusion, there can no longer be any doubt that the pendulum was used in the development of ancient metrology.

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Figure 12. Replica of The Parthenon of Ancient Greece.

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Roland A. Boucher (rolandfly@sbcglobal.net), Irvine, California.